Network Coding Capacity Bounds: Characterization and Computation

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July 20, 2011

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Introduction

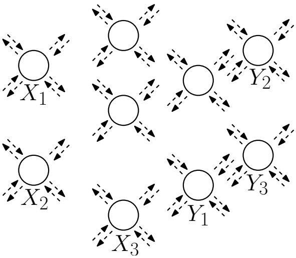


Figure: Multi-terminal.

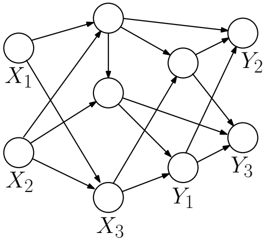


Figure: Point-to-point.

- Noisy, common channel
- Physical layer
- [Cover and Thomas 1991]

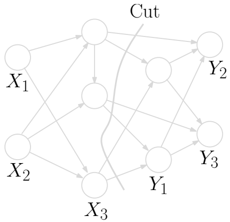
- Noise-free, Point-to-point channels
- Network layer
- Routing is suboptimal, Network coding
- [Ahlswede, Cai, Li and Yeung 2000]

Introduction

Important network information flow problems

- Characterization: Discover the limits of maximum feasible flow from "sources" to "sinks" through a network.
- Computation: Design efficient algorithms to compute the limits.

Characterization $\xrightarrow{\text{approaches}}$ Graph theoretic / Geometrical $\xrightarrow[\text{lead to}]{\text{if explicit}}$ Computation



- Intersection of sets of points in the Euclidean space

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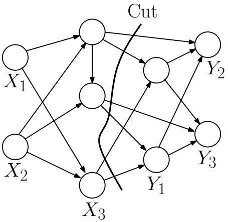


- Intersection of sets of points in the Euclidean space

Content

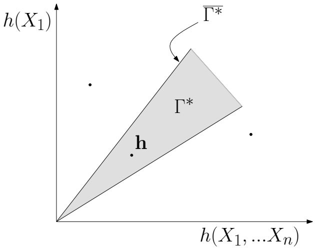
Content

Pseudo-variables and Pseudo-entropy

- Generalize random variables and entropy functions.
- Facilitates treatment of bounds and their interrelation.
- Let $X_{\mathcal{N}} = \{X_1, X_2, \ldots, X_n\}$ be a ground set associated with a real-valued function $h : 2^{X_{\mathcal{N}}} \mapsto \mathbb{R}$ defined on subsets of $X_{\mathcal{N}}$, with $h(\emptyset) = 0$.

- $X_1, X_2, \ldots, X_n$ - pseudo-variables
- h - pseudo-entropy

$$\mathbf{h} = [h(X_{\mathcal{A}}) : \mathcal{A} \subseteq \mathcal{N} \setminus \emptyset]^{\mathsf{T}}$$



Polymatroidal Pseudo-variables

- A function h over set $\mathcal{X}$ is called *polymatroidal* if, for all $\mathcal{A}, \mathcal{B} \subset \mathcal{X}$

$$
\begin{aligned}
h(\emptyset) &= 0 \\
h(\mathcal{A}) &\geq h(\mathcal{B}), \quad \text{if } \mathcal{B} \subseteq \mathcal{A} && \text{non-decreasing} \\
h(\mathcal{A}) + h(\mathcal{B}) &\geq h(\mathcal{A} \cup \mathcal{B}) + h(\mathcal{A} \cap \mathcal{B}) && \text{submodular}
\end{aligned}
$$

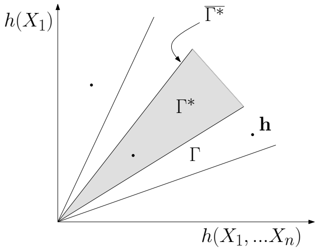
- Elemental inequalities - *minimal*
  [Yeung 1997]

$$
\begin{aligned}
& h(X_i | X_{\mathcal{N} \setminus \{i\}}) \geq 0, i \in \mathcal{N} \\
& I(X_i; X_j | X_{\mathcal{K}}) \geq 0, i \neq j, \mathcal{K} \subseteq \mathcal{N} \setminus \{i, j\}
\end{aligned}
$$

- $m = n + \binom{n}{2} 2^{n-2}$ inequalities.

$$\Gamma \triangleq \{\mathbf{h} : \mathbf{G}\mathbf{h} \geq \mathbf{0}\}$$

- $\mathbf{G}$ is an $m \times (2^n - 1)$ matrix
- $\mathbf{h}$ is a column $2^n - 1$ vector.



The region Γ for 3 pseudo-variables

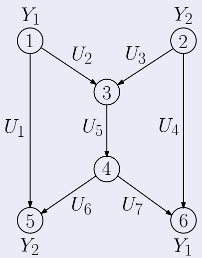
$$
\begin{aligned}
h(ABC) - h(BC) &\geq 0 \\
h(ABC) - h(AC) &\geq 0 \\
h(ABC) - h(AB) &\geq 0 \\
h(AC) + h(BC) - h(ABC) - h(C) &\geq 0 \\
h(A) + h(C) - h(AC) &\geq 0 \\
h(AB) + h(BC) - h(ABC) - h(B) &\geq 0 \\
h(B) + h(C) - h(BC) &\geq 0 \\
h(AC) + h(AB) - h(ABC) - h(A) &\geq 0
\end{aligned}
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 1 \\
1 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 1 & -1 \\
1 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 1 & -1 \\
0 & 1 & 1 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 1 & 1 & 0 & -1
\end{bmatrix}
\times
\begin{bmatrix}
h(A) \\ h(B) \\ h(C) \\ h(AB) \\ h(AC) \\ h(BC) \\ h(ABC)
\end{bmatrix}
\geq
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Geometric bounds

Definition

Given a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with sessions $\mathcal{S}$, source locations a and sink demands b, and a subset of pseudo-entropy functions $\Delta \subset \mathbb{R}^{2^{|\mathcal{S}|+|\mathcal{E}|}}$ on pseudo-variables $Y_{\mathcal{S}}, U_{\mathcal{E}}$, let $\mathcal{R}(\Delta)$ be the set of source pseudo-entropy tuples $\mathbf{h} = (h(Y_s), s \in \mathcal{S}) \in \mathbb{R}^{|\mathcal{S}|}$ for which there exists $h \in \Delta$ satisfying



$$h(Y_s : s \in \mathcal{S}) = \sum_{s \in \mathcal{S}} h(Y_s) \quad (\mathcal{C}_1)$$

$$h\left(U_e \mid \{Y_s : a(s) \to e\}, \{U_f : f \to e\}\right) = 0, e \in \mathcal{E} \quad (\mathcal{C}_2)$$

$$h\left(Y_s \mid U_e : e \to u\right) = 0, u \in b(s) \quad (\mathcal{C}_3)$$

$$h(U_e) \le C_e, e \in \mathcal{E} \quad (\mathcal{C}_4)$$

$$h(Y_s) \ge R_s, s \in \mathcal{S}$$

- Outer bounds $\mathcal{R}(\overline{\Gamma^*}), \mathcal{R}(\Gamma)$ [Yeung 2002]

$$\max \sum_{s \in \mathcal{S}} h(Y_s) \quad \text{subject to } h \in \mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_3 \cap \mathcal{C}_4 \cap \Gamma$$

The region Γ is of fundamental importance

- Weighted sum-rate LP bound

$$\text{maximize } \mathbf{w}^\mathsf{T}\mathbf{h} \text{ subject to } \begin{cases} \mathbf{G}\mathbf{h} & \geq 0 \\ \mathbf{c}_1^\mathsf{T}\mathbf{h} & = 0 \\ \mathbf{C}_2\mathbf{h} & = 0 \\ \mathbf{C}_3\mathbf{h} & = 0 \\ \mathbf{J}\mathbf{h} & \leq \mathbf{c}_4 \end{cases}$$

- Proving basic information inequalities: redundancy check
- Computer program: ITIP [Yeung and Yan]

$$\text{minimize } \mathbf{b}^\mathsf{T}\mathbf{h}, \text{ subject to } \begin{cases} \mathbf{G}\mathbf{h} & \geq 0 \\ \mathbf{C}\mathbf{h} & = 0 \end{cases}$$

- LP rate region

$$\text{project } \left\{ \begin{array}{c} \mathbf{Gh} \geq 0, \\ \mathbf{c}_1^\mathsf{T}\mathbf{h} = 0, \\ \mathbf{C}_2\mathbf{h} = 0, \\ \mathbf{C}_3\mathbf{h} = 0, \\ \mathbf{Jh} \leq \mathbf{c}_4 \end{array} \right\} \text{ on } [h(Y_s) : s \in \mathcal{S}]^\mathsf{T}$$

Computing is prohibitive since...

- Dimensions $= 2^n - 1$, constraints $> m = n + \binom{n}{2} 2^{n-2}$.
- Exhausts computational and memory resources
- Simplex Algorithm: Exponential worst case complexity in size
- Means, doubly exponential in n
- Projection: Fourier Motzkin elimination - triply exponential in n
- For example, $n \leq 5$ - pretty good
- $n = 6, 7$ - not bad
- $n = 8$ - Well, these may take a lot of time and space
- $n = 9$ - Well, these may blow up
- This is subject to the problem, the constraints and available computational and memory resources.

Warning: For large n,

Do not try these at home or lab!

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Graphical bounds

- Cut-set bound $\mathcal{R}_{CS}$ [Cover and Thomas 1991], [Borade 2002]
- Max-flow bound [Ahlswede, Cai, Li and Yeung 2000]
- Network sharing bound $\mathcal{R}_{NS}$ [Yan, Yang and Zhang 2006]
- Progressive *d*-separating edge-set bound $\mathcal{R}_{PdE}$ [Kramer and Savari 2006]

- A graphical concept - Information dominance [Harvey, Kleinberg and Lehman 2006]

Content

Functional Dependence Graphs

Definition (Functional Dependence Graph)

Let $\mathcal{X} = \{X_1, \ldots, X_N\}$ be a set of pseudo-variables with pseudo-entropy function h. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $|\mathcal{V}| = N$ is called a *functional dependence graph* for $\mathcal{X}$ if and only if for all $i = 1, 2, \ldots, N$

$$h\left(X_i \mid \{X_j : (j,i) \in \mathcal{E}\}\right) = 0.$$

$$h(X_2|X_1) = 0$$
$$h(X_3|X_1) = 0$$
$$h(X_4|X_2) = 0$$
$$h(X_5|X_3) = 0$$
$$h(X_1|X_4, X_5) = 0$$

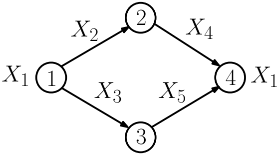


Figure: A network.

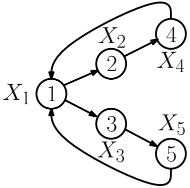


Figure: An FDG.

The Butterfly Network

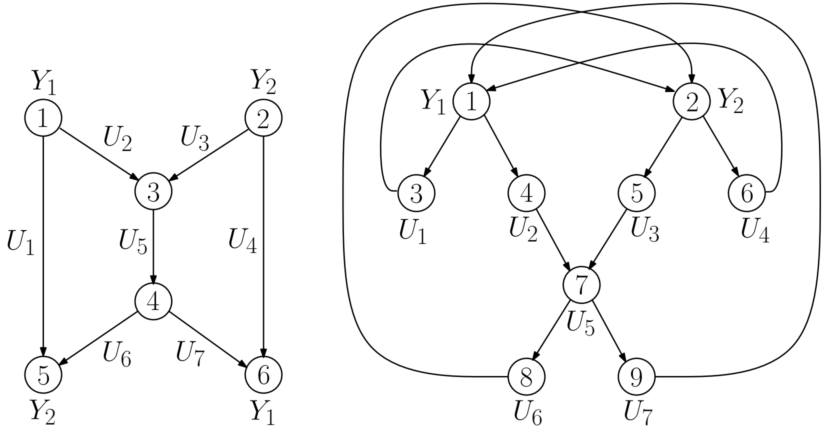


Figure: The butterfly network and FDG.

Functional Dependence Graphs

- FDG : More general than of [Kramer 1998, Chapter 2].
- No distinction between source and non-source pseudo-variables.
- Independence between sources is not must.
- Cyclic directed graphs.
- *Additional* functional dependence relationships.
- Holds for a wide class of objects.
- An FDG [Kramer 1998] is also FDG, but the converse is not true.

- FDG construction is based on local functional dependencies.
- Important to find implied functional dependencies, i.e., to find all sets $\mathcal{A}$ and $\mathcal{B}$ such that $h(\mathcal{A}\mathcal{B}) = h(\mathcal{A})$.

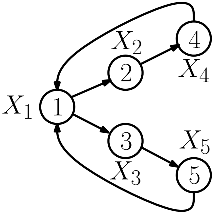
Functional dependence

Definition

For disjoint sets $\mathcal{A}, \mathcal{B} \subset \mathcal{V}$ we say $\mathcal{A}$ determines $\mathcal{B}$ in the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, denoted $\mathcal{A} \to \mathcal{B}$, if there are no elements of $\mathcal{B}$ remaining after the following procedure:
Remove all edges outgoing from nodes in $\mathcal{A}$ and subsequently remove all nodes and edges with no incoming edges and nodes respectively.

$$X_1 \longrightarrow \{X_2, X_3, X_4, X_5\}$$

$$X_2 \longrightarrow X_4$$



Functional dependence

Theorem

Let $\mathcal{G}$ be a functional dependence graph on the pseudo-variables $\mathcal{X}$ with polymatroidal pseudo-entropy function h. Then for disjoint subsets $\mathcal{A}, \mathcal{B} \subset \mathcal{V}$,

$$\mathcal{A} \to \mathcal{B} \implies h(\mathcal{B} \mid \mathcal{A}) = 0.$$

- An efficient graphical procedure to find implied functional dependencies.
- Chain rule and pseudo-entropy is non-increasing with respect to conditioning

Irreducible sets

Definition

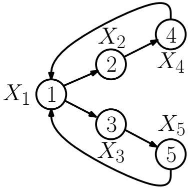
For a given set $\mathcal{A}$, let $\phi(\mathcal{A}) \subseteq \mathcal{V}$ be the set of nodes deleted by the procedure.

- Application: reduction of a set.
- i.e. If $\mathcal{C} = \mathcal{A}\mathcal{B}$ and $\mathcal{A} \rightarrow \mathcal{B} \implies h(\mathcal{C}) = h(\mathcal{A}\mathcal{B}) = h(\mathcal{A})$.
- Moreover, it also tells which sets are *irreducible*!

Definition (Irreducible set)

A set of nodes $\mathcal{B}$ in a functional dependence graph is *irreducible* if there is no $\mathcal{A} \subset \mathcal{B}$ with $\mathcal{A} \rightarrow \mathcal{B}$.

$$X_2 \longrightarrow X_4 \Longrightarrow h(X_2, X_4) = h(X_2)$$



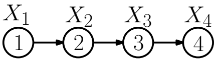
Maximal irreducible sets for acyclic graphs

Definition (Maximal irreducible sets)

An irreducible set $\mathcal{A}$ is *maximal* in an acyclic FDG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if $(\mathcal{V} \setminus \phi(\mathcal{A})) \setminus \mathrm{An}(\mathcal{A}) = \emptyset$, and no proper subset of $\mathcal{A}$ has the same property.

- Ancestral nodes $\mathrm{An}(\mathcal{A})$.
- Nodes deleted by the procedure $\phi(\mathcal{A}) \subseteq \mathcal{V}$.
- Every subset of a maximal irreducible set is irreducible. Conversely, every irreducible set is a subset of some maximal irreducible set.

X_1, X_2, X_3, X_4



Algorithm

- Let $\mathcal{G}$ be a topologically sorted.
- **AllMaxSetsA**$(\mathcal{G}, \{\})$ finds all maximal irreducible sets (recursive augmentation).

Algorithm

AllMaxSetsA$(\mathcal{G}, \mathcal{A})$

Require: $\mathcal{G} = (\mathcal{V}, \mathcal{E}), \mathcal{A} \subset \mathcal{V}$

  $\mathcal{B} \leftarrow (\mathcal{V} \setminus \phi(\mathcal{A})) \setminus \mathrm{An}(\mathcal{A})$

  if $\mathcal{B} \neq \emptyset$ **then**

    Output $\{$**AllMaxSetsA**$(\mathcal{G}, \mathcal{A} \cup \{b\}) : b \in \mathcal{B}\}$

  else

    Output $\mathcal{A}$

  end if

Lemma (Augmentation)

Let $A \subset \mathcal{V}$ in an acyclic FDG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Let $\mathcal{B} = \mathcal{V} \setminus \phi(\mathcal{A}) \setminus \mathrm{An}(\mathcal{A})$. Then $\mathcal{A} \cup \{b\}$ is irreducible for every $b \in \mathcal{B}$.

Maximal irreducible sets for cyclic graphs

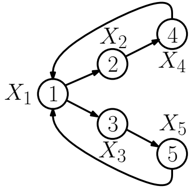
- The notion of a maximal irreducible set is modified as follows:

Definition

An irreducible set $\mathcal{A}$ is *maximal* in a cyclic FDG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if $\mathcal{V} \setminus \phi(\mathcal{A}) = \emptyset$, and no proper subset of $\mathcal{A}$ has the same property.

- Every subset of a maximal irreducible set is irreducible. Converse is not true.
- Lemma: All maximal irreducible sets have the same pseudo-entropy.

$$
\begin{aligned}
h(X_1, X_2, X_3, X_4, X_5) &= h(X_1) \\
&= h(X_2, X_3) = h(X_4, X_5) \\
&= h(X_2, X_5) = h(X_3, X_4)
\end{aligned}
$$



Interpretation: maximal irreducible sets are "information blockers" - information theoretic cuts.

Algorithm

- Algorithm finds all maximal irreducible sets that *do not* contain any node in $\mathcal{A}$.
- **AllMaxSetsC**$(\mathcal{G}, \{\})$ finds all maximal irreducible sets recursively.

Algorithm

AllMaxSetsC$(\mathcal{G}, \mathcal{A})$

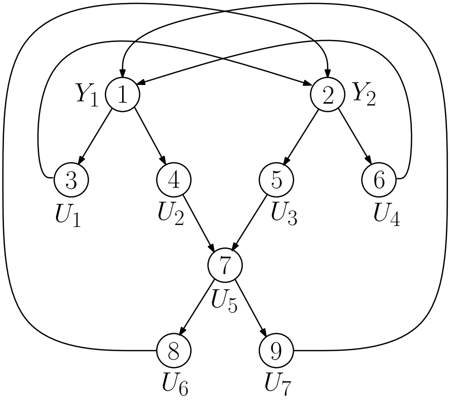
Require: $\mathcal{G} = (\mathcal{V}, \mathcal{E}), \mathcal{A} \subset \mathcal{V}$

```
if v ∉ φ(A^c \ {v}), ∀v ∈ A^c then
    Output A^c
else
    for all v ∈ A^c do
        if v ∈ φ(A^c \ {v}) then
            Output AllMaxSetsC(G, A ∪ {v})
        end if
    end for
end if
```

Maximal irreducible sets

$\mathcal{M} =$

$$\{\{1,2\},\{1,5\},\{1,7\},\{1,8\},$$
$$\{2,4\},\{2,7\},\{2,9\},\{3,4,5\},$$
$$\{3,4,8\},\{3,7\},\{3,8,9\},\{4,5,6\},$$
$$\{5,6,9\},\{6,7\},\{6,8,9\}\}.$$



Functional Dependence Bound

Theorem

Let $\mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4$ be given network coding constraint sets. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a functional dependence graph on the source and edge pseudo-variables $Y_{\mathcal{S}}, U_{\mathcal{E}}$ with pseudo-entropy function $h \in \mathcal{C}_2 \cap \mathcal{C}_3 \cap \mathcal{C}_4 \cap \Gamma$. Let $\boldsymbol{\mathcal{M}}$ be the collection of all maximal irreducible sets. Then

$$h(Y_{\mathcal{W}} | Y_{\mathcal{W}^c}) \leq \min_{\{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} \sum_{e \in \mathcal{A}} C_e,$$

where $\{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}$ is a maximal irreducible set and $Y_{\mathcal{W}}, Y_{\mathcal{W}^c} \subseteq Y_{\mathcal{S}}$. Moreover, if constraint $\mathcal{C}_1$ is given, that is, source pseudo-variables are independent, then

$$\sum_{s \in \mathcal{W}} h(Y_s) \leq \min_{\{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} \sum_{e \in \mathcal{A}} C_e.$$

Proof

Let $\mathcal{B} \in \boldsymbol{\mathcal{M}}$, then

$$\begin{aligned} h(Y_{\mathcal{S}}) &= h(\mathcal{B}) \\ h(Y_{\mathcal{W}}, Y_{\mathcal{W}^c}) &= h(U_{\mathcal{A}}, Y_{\mathcal{W}^c} : \{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}) \\ h(Y_{\mathcal{W}} | Y_{\mathcal{W}^c}) + h(Y_{\mathcal{W}^c}) &\leq \sum_{e \in \mathcal{A}, \{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} h(U_e) + h(Y_{\mathcal{W}^c}) \\ h(Y_{\mathcal{W}} | Y_{\mathcal{W}^c}) &\leq \sum_{e \in \mathcal{A}, \{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} h(U_e) \\ h(Y_{\mathcal{W}} | Y_{\mathcal{W}^c}) &\leq \sum_{e \in \mathcal{A}, \{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} C_e, \end{aligned}$$

If constraint $\mathcal{C}_1$ is given,

$$\sum_{s \in \mathcal{W}} h(Y_s) \leq \sum_{e \in \mathcal{A}, \{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} C_e.$$

Theorem: Functional Dependence Bound

$$\mathcal{R}_{FD} \triangleq \bigcap_{\mathcal{W} \subseteq \mathcal{S}} \left\{ \mathbf{h} \in \mathbb{R}_+^{2^{|\mathcal{S}|}-1} : h(Y_{\mathcal{W}} | Y_{\mathcal{W}^c}) \leq \min_{\{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} \sum_{e \in \mathcal{A}} C_e \right\}$$

When sources are independent:

$$\mathcal{R}_{FD} = \bigcap_{\mathcal{W} \subseteq \mathcal{S}} \left\{ \mathbf{h} \in \mathbb{R}_+^{|\mathcal{S}|} : \sum_{s \in \mathcal{W}} h(Y_s) \leq \min_{\{U_{\mathcal{A}}, Y_{\mathcal{W}^c}\} \in \boldsymbol{\mathcal{M}}} \sum_{e \in \mathcal{A}} C_e \right\}$$

Functional Dependence Bound

Example (Butterfly network with correlated sources)

The functional dependence bound is as follows using the maximal irreducible sets.

$$
\begin{aligned}
h(Y_1|Y_2) &\leq \min\{C_2, C_5, C_7\} \\
h(Y_2|Y_1) &\leq \min\{C_3, C_5, C_6\} \\
h(Y_1, Y_2) &\leq \min\{C_1 + C_5, C_4 + C_5, C_1 + C_2 + C_3 \\
&\qquad C_1 + C_2 + C_6, C_1 + C_6 + C_7, C_2 + C_3 + C_4 \\
&\qquad C_2 + C_3 + C_4, C_3 + C_4 + C_7, C_4 + C_6 + C_7\}
\end{aligned}
$$

Outline of contributions: correlated sources

- Bounds in terms of the regions $\overline{\Gamma^*}$ and Γ:

$$\mathcal{R}_{\text{cs}} \subset \mathcal{R}_{\text{cs}}(\overline{\Gamma^*}) \subset \mathcal{R}_{\text{cs}}(\Gamma)$$

- Proof similar to [Chapter 15, Yeung 2002]
- The bounds may not be tight - source correlation
- Demonstrated by example that $\mathcal{R}_{\text{cs}}(\Gamma)$ is loose.
- Improved bounds using auxiliary random variables
- Is it possible to incorporate the knowledge of probability distribution in the entropy space?
- Answer: yes, shown construction of auxiliary random variables describing the distribution of a given random variable.
- Approximate common information as auxiliary random variables.

Outline of contributions: correlated sources

- The functional dependence bound is a relaxation of $\mathcal{R}_{\text{cs}}(\Gamma)$.
- Other bounds (in fact, admissible regions):
  1. Multiple source single sink networks [Han 1980], [Barros and Servetto 2006]
  2. Multi source multi sink networks - all source are demanded by all sinks [Han 2011]

The functional dependence bound is the best known graphical bound for multi-source multi-sink networks with correlated sources.

Outline of contributions: independent sources

- Functional dependence bound for multicast networks with correlated sources
- A new bound for multicast networks with independent sources - dependence-independence bound $\mathcal{R}_{DI} \subseteq \mathcal{R}_{FD}$
- A new bound using information dominance $\mathcal{R}_{DI} = \mathcal{R}_{ID}$

Comparison and network constructions

$$\mathcal{R}_{FD} \subseteq \mathcal{R}_{CS}$$

$$\mathcal{R}_{FD} = \mathcal{R}_{NS}$$

$$\mathcal{R}_{DI} \subseteq \mathcal{R}_{PdE} \subseteq \mathcal{R}_{FD}$$

$$\mathcal{R}_{FD} \subsetneq \mathcal{R}_{CS}$$

$$\mathcal{R}_{DI} \subsetneq \mathcal{R}_{PdE} \Rightarrow \mathcal{R}_{DI} \subsetneq \mathcal{R}_{FD}$$

$$\Rightarrow \mathcal{R}_{DI} \subsetneq \mathcal{R}_{NS}$$

- Fine tuning of progressive *d*-separating edge-set bound.

The dependence-independence bound is the best known graphical bound for multi-source multi-sink networks with independent sources.

Content

Complexity Reduction

Recall the warning: For large n,

Do not try the problems at home or lab!

Contributions

- Characterized the feasible regions with significantly less inequalities

$$\left\{ \mathbf{h} : \begin{array}{rl} \mathbf{G}\mathbf{h} & \geq 0, \\ \mathbf{C}_2\mathbf{h} & = 0, \\ \mathbf{C}_3\mathbf{h} & = 0 \end{array} \right\} = \Gamma \cap \mathcal{C}_2 \cap \mathcal{C}_3 \iff \Upsilon_{\mathrm{cs}} \cap \Omega_{\mathrm{cs}} = \left\{ \mathrm{h} : \begin{array}{rl} \mathbf{M}_{\mathrm{cs}}\mathbf{h} & \geq 0, \\ \mathbf{K}_{\mathrm{cs}}\mathbf{h} & = 0, \\ \mathbf{L}_{\mathrm{cs}}\mathbf{h} & = 0 \end{array} \right\}$$

$$\left\{ \mathbf{h} : \begin{array}{rl} \mathbf{G}\mathbf{h} & \geq 0, \\ \mathbf{c}_1^\top\mathbf{h} & = 0, \\ \mathbf{C}_2\mathbf{h} & = 0, \\ \mathbf{C}_3\mathbf{h} & = 0 \end{array} \right\} = \Gamma \cap \mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_3 \iff \Upsilon \cap \Omega = \left\{ \mathrm{h} : \begin{array}{rl} \mathbf{M}\mathbf{h} & \geq 0, \\ \mathbf{K}\mathbf{h} & = 0, \\ \mathbf{L}\mathbf{h} & = 0 \end{array} \right\}$$

- Algorithms to *directly* generate the systems of reduced inequalities and equalities
- Upper bounds on the size of the systems of reduced inequalities and equalities

Complexity Reduction

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Contributions

- Characterized the feasible regions with significantly less inequalities

$$\left\{ \mathbf{h} : \begin{array}{rl} \mathbf{G}\mathbf{h} & \geq 0, \\ \mathbf{C}_2\mathbf{h} & = 0, \\ \mathbf{C}_3\mathbf{h} & = 0 \end{array} \right\} = \Gamma \cap \mathcal{C}_2 \cap \mathcal{C}_3 \Longleftrightarrow \Upsilon_{\mathrm{cs}} \cap \Omega_{\mathrm{cs}} = \left\{ \mathrm{h} : \begin{array}{rl} \mathbf{M}_{\mathrm{cs}}\mathbf{h} & \geq 0, \\ \mathbf{K}_{\mathrm{cs}}\mathbf{h} & = 0, \\ \mathbf{L}_{\mathrm{cs}}\mathbf{h} & = 0 \end{array} \right\}$$

$$\left\{ \mathbf{h} : \begin{array}{rl} \mathbf{G}\mathbf{h} & \geq 0, \\ \mathbf{c}_1^\top\mathbf{h} & = 0, \\ \mathbf{C}_2\mathbf{h} & = 0, \\ \mathbf{C}_3\mathbf{h} & = 0 \end{array} \right\} = \Gamma \cap \mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_3 \Longleftrightarrow \Upsilon \cap \Omega = \left\{ \mathrm{h} : \begin{array}{rl} \mathbf{M}\mathbf{h} & \geq 0, \\ \mathbf{K}\mathbf{h} & = 0, \\ \mathbf{L}\mathbf{h} & = 0 \end{array} \right\}$$

- Algorithms to *directly* generate the systems of reduced inequalities and equalities
- Upper bounds on the size of the systems of reduced inequalities and equalities

Reduction in Elemental Inequalities

- Remember: subject to the equality constraints.
- Notations: the set of all irreducible sets $\mathcal{K}$ and the set of all maximal irreducible sets $\mathcal{M} \subset \mathcal{K}$.

Lemma

Let $A \in \mathcal{V}$. If there exists $\mathcal{B} \in \mathcal{M}$ such that $\mathcal{B} \subseteq \mathcal{V} \setminus \{A\}, \mathcal{B} \neq \mathcal{S}$ then the elemental non-decreasing inequality $h(A|\mathcal{V} \setminus \{A\}) \geq 0$ can be replaced by

$$h(\mathcal{S}) - h(\mathcal{B}) = 0$$

where $\mathcal{B} \neq \mathcal{S} \in \mathcal{M}$. Otherwise, if no such $\mathcal{B}$ exists then the elemental non-decreasing inequality can be replaced by

$$h(\mathcal{S}) - h(\mathcal{B}) \geq 0$$

where $\mathcal{B} \subseteq \mathcal{V} \setminus \{A\}, \mathcal{V} \setminus \{A\} \subseteq \phi(\mathcal{B}), \mathcal{B} \in \mathcal{K} \setminus \mathcal{M}$ and $\mathcal{S} \in \mathcal{M}$.

- Modification of the inequalities, no reduction.

- Employing the notion of irreducible sets, there can be a significant reduction in the number of submodular inequalities.

Lemma

For $A, B \in \mathcal{V}$ and $\mathcal{C} \subseteq \mathcal{V} \setminus \{A, B\}$, if $\mathcal{C}$ is not irreducible then

$$I(A; B|\mathcal{C}) = I(A; B|\mathcal{B}) \tag{i}$$

where $\mathcal{B} \subset \mathcal{C} : \mathcal{C} \subseteq \phi(\mathcal{B}), \mathcal{B} \in \boldsymbol{\mathcal{K}}$.

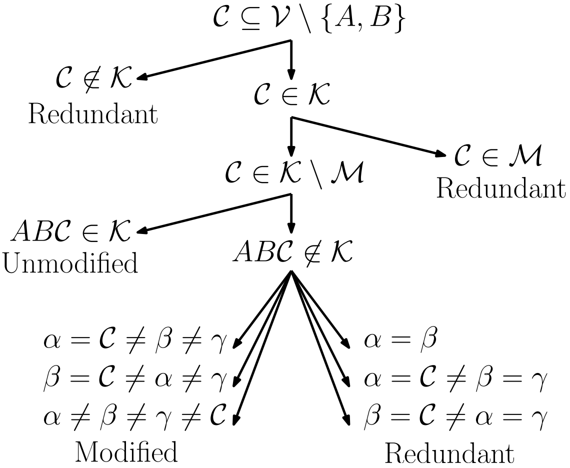
Lemma

For $A, B \in V$ and $\mathcal{C} \subseteq \mathcal{V} \setminus \{A, B\}$, let $\mathcal{C}$ be a maximal irreducible set then the elemental submodular inequality $I(A; B|\mathcal{C}) \geq 0$ is redundant. That is

$$I(A; B|\mathcal{C}) \geq 0, \mathcal{C} \in \boldsymbol{\mathcal{M}} \tag{ii}$$

are redundant inequalities.

- The difference in rationale for being redundant: (i) the inequality is same as another, (ii) the inequality is trivially zero.



Cases and their effect on elemental submodular inequalities
$I(A; B|\mathcal{C}) \geq 0$.

Corollary

For $A, B \in V$ and $\mathcal{C} \subseteq \mathcal{V} \setminus \{A, B\}$, let $AB\mathcal{C}$ be irreducible. If $AB\mathcal{C} \in \boldsymbol{\mathcal{M}}$ then the elemental submodular inequalities $I(A; B|\mathcal{C}) \geq 0$ can be replaced by

$$h(A\mathcal{C}) + h(B\mathcal{C}) - h(\mathcal{S}) - h(\mathcal{C}) \geq 0$$

where, $\mathcal{S} \in \boldsymbol{\mathcal{M}}$ is the set of all source pseudo-variables.

- Now, consider the case: $\mathcal{C}$ is irreducible but $AB\mathcal{C}$ is not irreducible. To study different cases within this case, we define the following set.

$$\boldsymbol{\mathcal{X}}(\mathcal{A}) \triangleq \{\alpha \subseteq \mathcal{A} : \alpha \in \boldsymbol{\mathcal{K}}, \mathcal{A} \subseteq \phi(\alpha)\}$$

- Note that the pseudo-entropy of the set of pseudo-variables $\mathcal{A}$ can be replaced by the pseudo-entropy of any set $\alpha \in \boldsymbol{\mathcal{X}}(\mathcal{A})$.

Lemma

For reducible set $AB\mathcal{C}$ and irreducible set $\mathcal{C}$, if there exists $\alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C})$ such that $\alpha = \beta$ then the elemental inequality $I(A;B|\mathcal{C}) \geq 0$ is redundant. That is,

$$I(A;B|\mathcal{C}) \geq 0 : \begin{array}{l} AB\mathcal{C} \notin \boldsymbol{\mathcal{K}}, \mathcal{C} \in \boldsymbol{\mathcal{K}}, \\ \alpha = \beta, \\ \alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C}) \end{array}$$

are redundant inequalities.

Lemma

For reducible sets ABC *and irreducible set* $\mathcal{C}$*, if there exists* $\alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C})$ *and* $\gamma \in \boldsymbol{\mathcal{X}}(AB\mathcal{C})$ *such that* $\alpha = \mathcal{C} \neq \beta = \gamma$ *or* $\beta = \mathcal{C} \neq \alpha = \gamma$ *then the elemental inequality* $I(A;B|\mathcal{C}) \geq 0$ *is redundant. That is,*

$$I(A;B|\mathcal{C}) \geq 0 : \begin{array}{l} AB\mathcal{C} \notin \boldsymbol{\mathcal{K}}, \mathcal{C} \in \boldsymbol{\mathcal{K}}, \\ \alpha = \mathcal{C} \neq \beta = \gamma, \\ \alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C}), \gamma \in \boldsymbol{\mathcal{X}}(AB\mathcal{C}) \end{array}$$

$$I(A;B|\mathcal{C}) \geq 0 : \begin{array}{l} AB\mathcal{C} \notin \boldsymbol{\mathcal{K}}, \mathcal{C} \in \boldsymbol{\mathcal{K}}, \\ \beta = \mathcal{C} \neq \alpha = \gamma \\ \alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C}), \gamma \in \boldsymbol{\mathcal{X}}(AB\mathcal{C}) \end{array}$$

are redundant inequalities since they can be written as trivial equalities.

Observe

- By proving inequalities to be redundant, we have reduced number of required inequalities.
- By modifying inequalities, we have reduced the dimension (the number of variables) of the required inequalities.

Algorithm

ReducedLPIneqCS$(\mathcal{V}, \boldsymbol{\mathcal{K}}, \boldsymbol{\mathcal{M}}, \mathcal{S} \in \boldsymbol{\mathcal{M}})$

Require: $\mathcal{V}, \boldsymbol{\mathcal{K}}, \boldsymbol{\mathcal{M}}, \mathcal{S} \in \boldsymbol{\mathcal{M}}$

```
I = ∅
for all A ∈ V do
    if ∃B ∈ K \ M : B ⊆ V \ {A} ⊆ φ(B) then
        I ← I ∪ {h(S) − h(B) ≥ 0}
    end if
end for
for all {A, B} ⊂ V, A ≠ B ≠ ∅ do
    for all C ∈ K \ M : A, B ∉ C do
        if {A, B, C} ∈ M then
            I ← I ∪ {h(AC) + h(BC) − h(S) − h(C) ≥ 0}
        else if {A, B, C} ∈ K \ M then
            I ← I ∪ {h(AC) + h(BC) − h(ABC) − h(C) ≥ 0}
        else if
```

$$\nexists \alpha \in \boldsymbol{\mathcal{X}}(A\mathcal{C}), \beta \in \boldsymbol{\mathcal{X}}(B\mathcal{C}), \gamma \in \boldsymbol{\mathcal{X}}(AB\mathcal{C}) : \begin{array}{l} \alpha = \beta = \gamma = \mathcal{C}, \\ \alpha = \mathcal{C} \neq \beta = \gamma, \\ \beta = \mathcal{C} \neq \alpha = \gamma \end{array}$$

```
        then
            Pick any α ∈ X(AC), β ∈ X(BC), γ ∈ X(ABC) and
            I ← I ∪ {h(α) + h(β) − h(γ) − h(C) ≥ 0}
        end if
    end for
end for
Output I
```

Algorithm

ReducedLPEqCS($\mathcal{V}, \boldsymbol{\mathcal{K}}, \boldsymbol{\mathcal{M}}, \mathcal{S} \in \boldsymbol{\mathcal{M}}$)

Require: $\mathcal{V}, \boldsymbol{\mathcal{K}}, \boldsymbol{\mathcal{M}} \subset \boldsymbol{\mathcal{K}}$

$\mathcal{J} = \emptyset$
for all $\mathcal{B} \in \boldsymbol{\mathcal{M}} \setminus \mathcal{S}$ **do**
  $\mathcal{J} \leftarrow \mathcal{J} \cup \{h(\mathcal{S}) - h(\mathcal{B}) = 0\}$
end for
for all $\mathcal{A}, \mathcal{B} \in \boldsymbol{\mathcal{K}} \setminus \boldsymbol{\mathcal{M}} : \mathcal{A} \neq \mathcal{B}, \mathcal{A} \cup \mathcal{B} \notin \boldsymbol{\mathcal{K}}$ **do**
  if $\mathcal{A} \cup \mathcal{B} \subseteq \phi(\mathcal{A})$ and $\mathcal{A} \cup \mathcal{B} \subseteq \phi(\mathcal{B})$ **then**
    $\mathcal{J} \leftarrow \mathcal{J} \cup \{h(\mathcal{A}) - h(\mathcal{B}) = 0\}$
  end if
end for
$\mathcal{J} \leftarrow \mathbf{ReducedRowEchelon}(\mathcal{J})$
for all $\mathcal{C} \subseteq \mathcal{V} : \mathcal{C} \notin \boldsymbol{\mathcal{K}}$ **do**
  Find a set $\mathcal{B}$ such that $\mathcal{B} \subset \mathcal{C} \subseteq \phi(\mathcal{B}), \mathcal{B} \in \boldsymbol{\mathcal{K}}$
  $\mathcal{J} \leftarrow \mathcal{J} \cup \{h(\mathcal{C}) - h(\mathcal{B}) = 0\}$
end for
Output $\mathcal{J}$

The new systems of inequalities and equalities

$$
\mathbf{M}_{\text{cs}}\mathbf{K}\mathbf{h} \geq 0 \triangleq \left\{ \begin{array}{rl} h(\mathcal{S}) - h(\mathcal{B}) \geq 0 & : \mathcal{S} \in \boldsymbol{\mathcal{M}}, \mathcal{B} \in \boldsymbol{\mathcal{K}} \setminus \boldsymbol{\mathcal{M}}, \\ & \;\; \mathcal{B} \subseteq \mathcal{V} \setminus \{A\} \subseteq \phi(\mathcal{B}_j), A \in \mathcal{V} \\ I(A; B|\mathcal{C}) \geq 0 & : AB\mathcal{C} \in \boldsymbol{\mathcal{K}} \\ h(\beta) - h(\gamma) \geq 0 & : \alpha = \mathcal{C} \neq \beta \neq \gamma \\ h(\alpha) - h(\gamma) \geq 0 & : \beta = \mathcal{C} \neq \alpha = \gamma \\ h(\alpha) + h(\beta) - h(\gamma) - h(\mathcal{C}) \geq 0 & : \alpha \neq \beta \neq \gamma \neq \mathcal{C} \end{array} \right\}
$$

$$
\mathbf{K}_{\text{cs}}\mathbf{h} = 0 \triangleq \left\{ \begin{array}{rl} h(\mathcal{S}) - h(\mathcal{B}) = 0 & : \mathcal{B} \in \boldsymbol{\mathcal{M}} \setminus \mathcal{S} \\ h(\mathcal{B}) - h(\mathcal{C}) = 0 & : \mathcal{B}, \mathcal{C} \in \boldsymbol{\mathcal{K}} \setminus \boldsymbol{\mathcal{M}}, \mathcal{B} \cup \mathcal{C} \notin \boldsymbol{\mathcal{K}}, \\ & \;\; \mathcal{B} \cup \mathcal{C} \subseteq \phi(\mathcal{B}), \mathcal{B} \cup \mathcal{C} \subseteq \phi(\mathcal{C}) \end{array} \right\}
$$

$$
\mathbf{L}_{\text{cs}}\mathbf{h} = 0 \triangleq \left\{ h(\mathcal{C}) - h(\mathcal{B}) = 0 : \;\; \mathcal{C} \notin \boldsymbol{\mathcal{K}}, \mathcal{B} \in \boldsymbol{\mathcal{K}}, \mathcal{B} \subset \mathcal{C} \subseteq \phi(\mathcal{B}) \;\; \right\}
$$

- Define

$$\Upsilon_{\mathrm{cs}} \triangleq \{\mathbf{h} : \mathbf{M}_{\mathrm{cs}}\mathbf{h} \geq 0\}$$
$$\Omega_{\mathrm{cs}} \triangleq \{\mathbf{h} : \mathbf{K}_{\mathrm{cs}}\mathbf{h} = 0, \mathbf{L}\mathbf{h} = 0\}$$

- The intersection

$$\Upsilon_{\mathrm{cs}} \cap \Omega_{\mathrm{cs}} = \{\mathbf{h} : \mathbf{M}_{\mathrm{cs}}\mathbf{h} \geq 0, \mathbf{K}_{\mathrm{cs}}\mathbf{h} = 0, \mathbf{L}\mathbf{h} = 0\}$$

Theorem

The region $\Gamma \cap \mathcal{C}_2 \cap \mathcal{C}_3$ and the region $\Upsilon_{\mathrm{cs}} \cap \Omega_{\mathrm{cs}}$ are the same.

- We also characterize the reduced system of inequalities and equalities when source independence is given:

Theorem

The region $\Gamma \cap \mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_3$ and the region $\Upsilon \cap \Omega$ are the same.

Upper bounds on the size

Lemma

Given the set of all maximal irreducible sets $\mathcal{M}$ and the set of all irreducible sets $\mathcal{K} \supset \mathcal{M}$, the number of inequalities describing the regions Υ_{cs} and Υ is upper bounded by

$$n + \binom{n}{2} |\mathcal{K} \setminus \mathcal{M}|$$

and the number of dimensions describing the region Υ is upper bounded by

$$|\mathcal{K} \setminus \mathcal{M}|.$$

Lemma

The number of equalities and dimensions describing regions Ω_{cs} and Ω is upper bounded by 2^n.

The problems reformulated

- Weighted sum-rate LP bound

$$\text{maximize } \mathbf{w}^\mathsf{T}\mathbf{h} \text{ subject to } \begin{cases} \mathbf{M}_{\mathsf{cs}}\mathbf{h} \geq 0 \\ \mathbf{J}\mathbf{h} \leq \mathbf{c}_4 \end{cases}$$

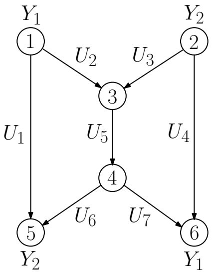
- Redundancy check (ITIP)

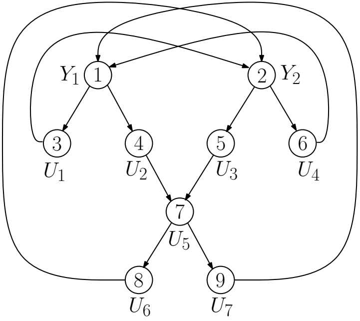
$$\text{minimize } \mathbf{b}^\mathsf{T}\mathbf{h} \text{ subject to } \mathbf{M}\mathbf{h} \geq 0$$

- Projection

$$\text{project } \begin{Bmatrix} \mathbf{M}\mathbf{h} \geq 0 \\ \mathbf{J}\mathbf{h} \leq \mathbf{c}_4 \end{Bmatrix} \text{ on } [h(Y_s) : s \in \mathcal{S}]^\mathsf{T}$$

Butterfly Network





$$
\begin{aligned}
\boldsymbol{\mathcal{M}} &= \{\{1,2\},\{1,5\},\{1,7\},\{1,8\},\{2,4\},\{2,7\},\{2,9\},\{3,4,5\},\\
&\quad \{3,4,8\},\{3,7\},\{3,8,9\},\{4,5,6\},\{5,6,9\},\{6,7\},\{6,8,9\}\}\\
\boldsymbol{\mathcal{K}} \setminus \boldsymbol{\mathcal{M}} &= \{\emptyset,\{1\},\{2\},\{13\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{1,6\},\{1,9\},\\
&\quad \{2,3\},\{2,8\},\{3,4\},\{3,5\},\{3,6\},\{3,8\},\{3,9\},\{4,5\},\{4,6\},\\
&\quad \{4,7\},\{4,8\},\{4,9\},\{5,6\},\{5,7\},\{5,8\},\{5,9\},\{6,8\},\{6,9\},\\
&\quad \{8,9\},\{3,4,6\},\{3,4,9\},\{3,5,6\},\{3,5,9\},\{4,6,8\},\{4,8,9\},\\
&\quad \{5,6,8\},\{5,8,9\}\}
\end{aligned}
$$

Butterfly Network

The Original LP

- Dimensions: $2^9 - 1 = 511$
- Constraints: $13 + 9 + \binom{9}{2}2^7 = 4630$

The reduced LP

- Dimensions: $|\mathcal{K} \setminus \mathcal{M}| = 39$
- The upper bound on the number of inequalities: $54 + 7 + \binom{9}{2}39 = 1465$
- The number of inequalities generated by the algorithm: 844
- The total number of inequalities for the LP bound computation: 851

Butterfly Network

- The size of the constraints matrix is reduced from 4630×511 to 851×39.
- 92% reduction in the number of dimensions,
- 81% reduction in the number of constraints and
- 98% reduction in the size of the problem.

Recent result

Compact formulation of polymatroid axioms for random variables with conditional independencies

Content

Conclusion

- The notion of irreducible sets
- New better bounds - (i) correlated sources, (ii) independent sources
- Algorithms
- Comparison
- Complexity reduction of linear programming problems
- The complexity reduction approach is applicable to many problems involving linear information inequalities and equalities

Publications

S. Thakor, A. Grant and T. Chan.
Network coding capacity: A functional dependence bound.
in *International Symposium on Information Theory*, pp.263–267, Jul 2009.

S. Thakor, T. Chan and A. Grant.
Bounds for network information flow with correlated sources.
in *Australian Communications Theory Workshop*, pp.43–48, Jan 2011.

S. Thakor, A. Grant and T. Chan.
On complexity reduction of the LP bound computation and related problems.
accepted in *International Symposium on Network Coding*, Jul 2011.

S. Thakor, A. Grant and T. Chan.
Compact representation of polymatroid axioms for random variables with conditional independencies.
submitted to *Information Theorey Workshop*, 2011.

Journal papers in preparation

- On capacity bounds
- On complexity reduction

Q & A

To bird and *butterfly*
it is unknown, this flower here:
the autumn sky.
– Bashō, 17th Century